1. A company supplies goods to three customers, who each require 30 units. The company has two warehouses. Warehouse 1 has 40 units available, and warehouse 2 has 30 units available. The costs of shipping are listed in the following table. There is a penalty for each unmet customer unit of demand: With customer 1, a penalty cost of $90 is incurred; with customer 2, $80; and with customer 3, $110. Formulate a balanced transportation problem to minimize the sum of shortage and shipping costs. Please show your answer in a transportation tableau. (25%)

| From | To | | |
| --- | --- | --- | --- |
| | Customer 1 | Customer 2 | Customer 3 |
| Warehouse 1 | $15 | $35 | $25 |
| Warehouse 2 | $10 | $50 | $40 |

2. Referring to Problem 1, suppose that extra units could be purchased and shipped to either warehouse for a total cost of $100 per unit and that all customer demand must be met. Formulate a balanced transportation problem to minimize the sum of purchasing and shipping costs. Please show your answer in a transportation tableau. (25%)

3. Use the northwest corner method to find a basic feasible solution for problem 2. (25%)

4. Use the simplex algorithm to find the optimal solution to the following LP: (25%)

$$\text{Min } z = 4x_1 - x_2$$
$$\text{s.t.} \quad 2x_1 + x_2 \leq 8$$
$$x_2 \leq 5$$
$$x_1 - x_2 \leq 4$$
$$x_1, x_2 \geq 0$$