註:本次考試　不可以參考自己的書籍及筆記；不可以使用字典；不可以使用計算器。

1. As for the homogeneous system

$$-2x_3 + 7x_5 = 0$$
$$2x_1 + 4x_2 - 10x_3 + 6x_4 + 12x_5 = 0$$
$$2x_1 + 4x_2 - 5x_3 + 6x_4 - 5x_5 = 0$$

(a) (10%) Solve the system by Gauss-Jordan elimination.
(b) (10%) Determine a basis for and the dimension of the solution space.

2. For the following matrix A

$$A = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$$

Find

(a) (5%) the characteristic equation of A

(b) (5%) the eigenvalues of A

(c) (10%) the eigenvectors corresponding to each of the eigenvalues of A.

(d) (5%) the eigenvalues of A^{10}

3. (15%) Solve the general solution for the following differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 8e^x$$

4. (20%) Solve the general solution for the following differential equation

$$\frac{d^2y}{dt^2} + 4y = -t\sin(2t)$$

5. (20%) Find Fourier series expansion for the following function:

$$f(x) = x \quad \text{for} \quad -\pi \le x \le \pi$$