

科目名稱: 微積分 系列: 各系所

共一頁

註: 本次考試不可參考書籍及筆記

不可使用字典

不可使用計算機

1. Evaluate the limits:(每小題6分)

(a) $\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x - 3}$.

(b) $\lim_{x \rightarrow 3} \frac{\sqrt{x} - \sqrt{3}}{x - 3}$.

(c) $\lim_{x \rightarrow 3^-} \left(\frac{x^2 + 9}{x - 3} \right)$.

2. Find the derivative (導函數) $\frac{dy}{dx}$ of the followings:(每小題6分)

(a) $y = e^{x^2}(x + 1)$.

(b) $y = (x^{1/2} - 1)^{1/2}$.

(c) $x + 2x^3y + y^2 = 3$.

3. Find the equation of the tangent line (切線) to the curve $y = \frac{x - 1}{x + 2}$ at $(4, \frac{1}{2})$. (10分)

4. Find all extrema (極值) for $y = x - 2 \ln x$ on the interval $[1, 3]$. (12分)

5. Evaluate the following integrals:(每小題6分)

(a) $\int \left(\frac{x}{5} - \frac{3}{x^2} + 5x^4 \right) dx$.

(b) $\int x(x^2 + 9)^5 dx$.

(c) $\int_1^t x \ln x dx$.

6. Find the region of convergence of the power series (12分)

$$\frac{(x + 2)^2 \ln 2}{2 \times 9} + \frac{(x + 2)^3 \ln 3}{3 \times 27} + \frac{(x + 2)^4 \ln 4}{4 \times 81} + \dots$$

7. Find $\int_R \int \frac{y}{1 + x} dA$ where R is the region bounded by $y = 0$, $y = \sqrt{x}$, and $x = 4$. (12分)