大同大學 100 學年度(寒)轉學入學考試試題

考試科目:微積分　　所別:各系所　　第全頁

註:本次考試不可以參考自己的書籍及筆記；不可以使用字典；不可以使用計算器。

1. Find the derivatives of the following functions. (12%)

(a) $f(x)=x\sin(2x)$　　(b) $g(x)=\dfrac{x}{x^2+1}$

2. Evaluate the following limits. (18%)

(a) $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x-2\sin x}$　　(b) $\lim\limits_{x\to 0}\dfrac{xe^x}{\ln(2x+1)}$　　(c) $\lim\limits_{x\to\infty}\ln\left(\frac{1}{x}+e^{\frac{1}{x}}\right)^x$

3. Evaluate the following integrals. (18%)

(a) $\int \cos x\sqrt{1+\sin x}\,dx$　　(b) $\int x^3\ln x\,dx$　　(c) $\int\dfrac{3x-4}{x^2-2x}\,dx$

4. Determine whether the series is convergent or divergent. (16%)

(a) $\sum\limits_{n=1}^{\infty}\dfrac{1}{n}\cos n\pi$　　(b) $\sum\limits_{n=1}^{\infty}\sin\left(\frac{n}{n+1}\right)$

5. Find an equation of the tangent plane to the surface $2x^3y^2+y^3+z^2=0$ at the point $(-1,1,1)$. (8%)

6. Let $f(x,y)=\int_x^{y^2}\sqrt{1+t^5}\,dt$. Find $f_x(x,y)$ and $f_y(x,y)$ (8%)

7. Find the area of the region that is inside $r=1+\cos\theta$ and is outside $r=\cos\theta$. (10%)

8. Evaluate $\iiint_Q\dfrac{1}{\sqrt{9-x^2-y^2-z^2}}\,dxdydz$, where $Q=\{(x,y,z):x^2+y^2+z^2\le 9\}$. (10%)