

# 大同大學九十四學年度轉學考試試題

考試科目：微積分

第1頁，共1頁

註：本次考試不可以參考自己的書籍及筆記；不可以使用字典；不可以使用計算器。

一、填充題 80%

1. Let  $a_1 = 4, a_{n+1} = \frac{a_n}{2} + \frac{1}{a_n}, n \in N$ , and if  $\{a_n\}$  converges. Then  $\lim_{n \rightarrow \infty} a_n =$  \_\_\_\_\_

2. Let  $f(x) = (1+x)^x$ , then  $f'(2) =$  \_\_\_\_\_

3. Let  $f(x) = 2x^3 + x - 17$ , then  $(f^{-1})'(1) =$  \_\_\_\_\_

4.  $\int \tan x \, dx =$  \_\_\_\_\_

5.  $\int_0^1 \frac{3x+5}{x^2+3x+2} \, dx =$  \_\_\_\_\_

6. The equation of the tangent line to  $y = \tan x$  at  $x = 0$  is \_\_\_\_\_

7.  $\frac{1}{2} + \frac{1}{1 \times 2} + \frac{1}{2^2} + \frac{1}{2 \times 3} + \frac{1}{2^3} + \frac{1}{3 \times 4} + \dots + \frac{1}{2^n} + \frac{1}{n(n+1)} + \dots =$  \_\_\_\_\_

8. The arc length of  $C: x(t) = \cos t, y(t) = \sin t, z(t) = t, 0 \leq t \leq 2\sqrt{2}\pi$  is \_\_\_\_\_

9. The convergence set of the series  $\sum_{n=0}^{\infty} \left(-\frac{1}{3}\right)^n (x-4)^n$  is \_\_\_\_\_

10. Let  $\Omega = \{(x, y) \mid x^2 + y^2 \leq 1\}$ , then  $\iint_{\Omega} \sqrt{x^2 + y^2} \, dx \, dy =$  \_\_\_\_\_

二、計算題 20%

1. Let  $f(n) = \left( \int_0^{\frac{\pi}{2}} \cos^n \theta \, d\theta \right) \left( \int_0^{\frac{\pi}{2}} \cos^{n-1} \theta \, d\theta \right), n \in N$ , find  $f(100)$ .

2. Find the local extrema of  $f(x, y) = 2x^2 + xy + y^2 - 2x + 3y + 8$ .