

大同大學 九十三年 學年度 轉學考試 試題

考試科目：微積分

第一頁，共一頁

註：本次考試不可以參考自己的書籍及筆記； 不可以使用字典； 不可以使用計算器。

一、填充題 80%

1. Let $P(x) = x^{10} + x^9 + \cdots + x$, then $\lim_{x \rightarrow 0} \frac{\sqrt{1+P(x)} - 1}{x} = \underline{\hspace{2cm}}$.

2. Let $a > b > 0$, then the radius of convergence of the power series

$$\sum_{n=1}^{\infty} \left(\frac{a^n}{n} + \frac{b^n}{n} \right) x^n \text{ is } \underline{\hspace{2cm}}.$$

3. $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{n}{n^2 + k^2} = \underline{\hspace{2cm}}$.

4. Let $F(x) = \int_0^{x^2} \frac{1}{1 + \sin^2 t} dt$, then $F'(1) = \underline{\hspace{2cm}}$.

5. (1) $\int_0^{\pi/2} \frac{\sin x \cos x}{1 + \sin^2 x} dx = \underline{\hspace{2cm}}$. (2) $\int_0^1 \frac{3x+5}{x^2+3x+2} dx = \underline{\hspace{2cm}}$.

6. Let $\Omega = \{(x, y) \mid x^2 + y^2 \leq 1\}$, then $\iint_{\Omega} \sqrt{x^2 + y^2} dx dy = \underline{\hspace{2cm}}$.

7. The area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\underline{\hspace{2cm}}$.

8. If $f(x, y) = \tan^{-1} \frac{x}{y}$, then $f_x + f_y = \underline{\hspace{2cm}}$.

9. The equation of the tangent plane for the surface $z = e^x \cos y$ at $(1, 0, e)$

is $\underline{\hspace{2cm}}$.

二、計算題 20%

1. Evaluate the limit $\lim_{n \rightarrow \infty} [(n^{2003} + n^{2004})^{1/2004} - n]$.

2. Find the extrema, if any, of the function

$$f(x, y) = 3x^3 + y^2 - 9x + 4y - 4.$$