

大同大學 九十一 學年度 轉學考試 試題

考試科目：工程數學 系別：化學工程學系

級別：三年級 第 1 頁，共 1 頁

註：本次考試不可以參考自己的書籍及筆記； 不可以使用字典； 不可以使用計算器。

Problem 1~5 solve the ODEs.

1. $y'+2y = x$ (5%)
2. $x^2 y'' - 9xy' + 25y = 0$ (5%)
3. $y'' + \frac{1}{x} y' + (1 - \frac{1}{4x^2}) y = 0, y_1 = \frac{1}{\sqrt{x}} \cos(2x), x > 0$ (10%)
4. $y'' + 2y' - 3y = 13 \cos(2x)$ (10%)
5. $y'' + 4y' + 4y = \begin{cases} 1, & 0 < t < 2 \\ 0, & t > 2 \end{cases}, y(0) = 1, y'(0) = 2$ (10%)
6. Determine whether the function $f(x) = |x|, -\pi < x < \pi$, is even, odd, or neither and expand in an appropriate Fourier cosine series, Fourier sine series or Fourier series. (10%)

$$ku_{xx} = u_t, 0 < x < L, t > 0$$

7. Solve the problem : $u(0, t) = 0, u(L, t) = 0$ (25%)
 $u(x, 0) = \begin{cases} 1, & 0 < x < L/2 \\ 0, & L/2 < x < L \end{cases}$

Handwritten notes for problem 7:
 $u(x, 0) = \begin{cases} 1, & 0 < x < L/2 \\ 0, & L/2 < x < L \end{cases}$
 $f(x) = \begin{cases} 1, & 0 < x < L/2 \\ 0, & L/2 < x < L \end{cases}$
 $f(x) = \frac{1}{2} - \frac{1}{2} \cos(\frac{2\pi x}{L}) + \frac{1}{4} \cos(\frac{4\pi x}{L}) - \frac{1}{8} \cos(\frac{6\pi x}{L}) + \dots$

8. Suppose heat is lost from the lateral surface of a thin rod of length L into a surrounding medium at temperature zero. If the linear law of heat transfer applies, then the heat equation takes on the form $ku_{xx} - hu = u_t, 0 < x < L, t > 0$, h a constant. If the initial temperature is f(x) throughout and the ends $x=0$ and $x=L$ are insulated. Write all the possible **initial and boundary conditions**. (15%)
9. Find $\vec{\nabla} \cdot F$ for $F = xy^2 + 3x^2 - z^3$ and $\vec{\nabla} \cdot \vec{F}$ for the vector $\vec{F} = x^2y \vec{i} + xy^2 \vec{j} + 2xyz \vec{k}$. (10%)

Handwritten notes for problem 9:
 $\vec{\nabla} \cdot F = \frac{\partial}{\partial x}(xy^2 + 3x^2 - z^3) + \frac{\partial}{\partial y}(xy^2 + 3x^2 - z^3) + \frac{\partial}{\partial z}(xy^2 + 3x^2 - z^3)$
 $= y^2 + 6x - 3z^2 + 2xy + 0 + 0 = y^2 + 6x + 2xy - 3z^2$
 $\vec{\nabla} \cdot \vec{F} = \frac{\partial}{\partial x}(x^2y) + \frac{\partial}{\partial y}(xy^2) + \frac{\partial}{\partial z}(2xyz)$
 $= 2xy + 2xy + 2xy = 6xy$

Handwritten notes for problem 9:
 $\vec{\nabla} \cdot \vec{F} = \frac{\partial}{\partial x}(x^2y) + \frac{\partial}{\partial y}(xy^2) + \frac{\partial}{\partial z}(2xyz)$
 $= 2xy + 2xy + 2xy = 6xy$