1. (15%) Solve $2x - y\sin(xy) + (3y^2 - x\sin(xy))y' = 0$

2. (15%) Solve y'' + y' + y = $6\cos(2x) + 3e^{-2x}$; y(0) = 2, y'(0) = 0

3. (15%) Solve $9x^2 y'' + 9xy' + (4x^{2/3} - 16)y = 0$　Hint: set $z = 2x^{1/3}$

4. (20%) Solve the BVP

$$\frac{\partial^2 y}{\partial t^2} = 4\frac{\partial^2 y}{\partial x^2} \quad \text{for x>0, t>0}$$

$$y(0,t) = \sin(t) \quad \text{for t > 0}$$

$$y(x,0) = 0, \quad \frac{\partial y}{\partial t}(x,0) = e^{-x} \quad \text{for x>0}$$

5. (20%) Solve the BVP with both T and L are constants.

$$\frac{\partial u}{\partial t} = 9\frac{\partial^2 u}{\partial x^2} \quad \text{for 0 < x<L, t>0}$$

$$u(0,t) = T, \quad u(L,t) = 0 \quad \text{for t} \geq 0$$

$$u(x,0) = 0 \quad \text{for } 0 \leq x \leq L$$

Hint: $\int_0^2 (x^3 - 4x)\sin(n\pi x/2)dx = 96(-1)^n / n\pi^3$

$\int_0^L T(1 - \xi/L)\sin(n\pi\xi/L)d\xi = TL/n\pi$

6. (15%) A 300-L tank initially contains 100 liter of glucose solution in which 10 kg of glucose have been dissolved. Beginning at time zero, a mixture containing 0.3 kg of glucose per liter is added into tank at the rate of 20 liter per minute, and a different stream containing pure water only is also added into tank at the rate of 10 liter per minute. Meanwhile, a stream is poured out from tank at the rate of 30 liter per minute.

(1) Determine how much glucose is in the tank at any time t?

(2) How much glucose will in the tank at steady state?